Spin phase diagram of the $\nu_e = 4/11$ composite fermion liquid

Arkadiusz Wójs,[1,2] George Simion,[2] and John J. Quinn[2]
[1] *Institute of Physics, Wrocław University of Technology, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland*
[2] *Department of Physics, University of Tennessee, Knoxville, TN 37996, USA*

Spin polarization of the "second generation" $\nu_e = 4/11$ fractional quantum Hall state (corresponding to an incompressible liquid in a one-third-filled composite fermion Landau level) is studied by exact diagonalization. Spin phase diagram is determined for GaAs structures of different width and electron concentration. Transition between the polarized and partially unpolarized states with distinct composite fermion correlations is predicted for realistic parameters.

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I. INTRODUCTION

There has been considerable speculation about the nature of "second-generation" incompressible quantum liquid (IQL) states observed recently by Pan *et al.*[1] Their incompressibility depends on spin and charge dynamics of the fractionally charged Laughlin quasiparticles (QPs).[2]

Pan's experiment employed the fractional quantum Hall (FQH) effect,[3] a non-perturbative interaction many-body phenomenon, dependent on magnetic quantization of the two-dimensional single-electron energy spectrum into massively degenerate Landau levels (LLs).[4] It coincides with the formation of electron IQLs and thus occurs at the particular fractional values of the LL filling factor, defined as $\nu_e = 2\pi\varrho_e\lambda^2$ (where ϱ_e is the electron concentration and $\lambda = (\hbar c/eB)^{1/2}$ is the magnetic length).

The emergence of IQLs is explained as follows by the composite fermion (CF) theory.[5] Electrons partially filling the lowest LL are said to capture $2p$ magnetic flux quanta $\phi_0 = hc/e$ and become (weakly interacting) CFs moving in a reduced effective magnetic field, corresponding to a higher effective CF filling factor $\nu_{\rm CF}$. The most prominent IQL sequence at $\nu_e = s(2ps \pm 1)^{-1}$ (with s and p being a pair of integers) corresponds to $\nu_{\rm CF} = s$, i.e., to the integral quantum Hall effect of the CFs.

However, not all IQLs found in the lowest LL can be explained in this way. Recently, Pan *et al.*[1] observed the FQH effect at $\nu_e = \frac{4}{11}$, corresponding to $\nu_{\rm CF} = \frac{4}{3}$, i.e., to a partal filling of a CF-LL. This discovery demonstrated that CFs, like electrons, can form IQLs. The origin of incompressibility of Pan's correlated CF liquid (also called a "second-generation" FQH state) has been vigorously studied for the last three years.[6,7,8,9,10,11,12] However, some of even most fundamental questions remain controversial.

The subject of this paper is polarization of the $\nu_e = \frac{4}{11}$ state. It is largely motivated by the wealth of theory of spin dynamics in the "first-generation" FQH states.[13,14,15,16,17,18,19,20] However, our main goal is to extend the work of Chang *et al.*[21] and directly address Pan's experimental results in tilted magnetic fields[1] which indicated ferromagnetic order. In the CF picture, this corresponds to a completely filled lowest CF-LL ($0\uparrow$) and a $\frac{1}{3}$-filled first excited CF-LL with the same spin ($1\uparrow$). Since the Laughlin $\nu = \frac{1}{3}$ state in CF-LL$_1$ was earlier ruled out[22] based on the form of short-range CF–CF interaction pseudopotential, the explanation for the observed incompressibility must be different. This distinction makes the polarized $\nu_e = \frac{4}{11}$ state an object of intense investigation.[23] Although several ideas were formulated (e.g., CF pairing[9,11]), neither an analytic CF wavefunction nor an intuitive understanding for the incompressibility has been reached. A partially unpolarized state was also proposed,[24] with the $\nu = \frac{1}{3}$ filling of the lowest CF-LL with reversed spin ($0\downarrow$). In contrast to the polarized state and due to a different form[25] of CF–CF interaction in CF-LL$_0$, it is expected to be a Laughlin CF liquid. However, this state has not yet been observed in experiment.

Let us summarize this remarkable situation as follows: The polarized state has been observed but it is not well understood, and the unpolarized state has not been observed but it appears to be much easier to understand. In this paper we calculate the single-particle and correlation energies in these two competing CF states, depending on the experimenetally controlled parameters (electron layer width, concentration, and magnetic field). The main result is the spin phase diagram, from which we predict a spin transition at $\nu_e = \frac{4}{11}$, induced e.g. by an additional electric field narrowing the electron layer. Suggested experimental demonstration of this transition would shine more light on the role played by spin of correlated CFs.

II. NUMERICAL MODEL

The calculations were done in Haldane's spherical geometry,[26] convenient for the numerical studies of incompressible quantum liquids with short-range correlations. To model an extended (planar) 2D system of interacting particles filling a fraction ν of a degenerate LL, their finite number N is considered within a shell of appropriate angular momentum l and degeneracy $g = 2l+1$ (containing states with different angular momentum projections, $|m| \le l$). The assignment of the filling factor ν to a finite system (N, g) is not trivial. It requires identifying dependence $g = \nu^{-1}N + \gamma$ which defines a series of finite systems representing an infinite state ν (here, the

"shift" γ is independent of N but it depends on the form of correlations, i.e., in particular on ν).

In the original formulation,[26,27] these l-shells represent LLs of a charged particle confined to a surface of a sphere of radius R, with the normal magnetic field B produced by a Dirac monopole of strength $2Q = 4\pi R^2 B/\phi_0$. Specifically, the nth LL on a plane (called LL_n; with $n \geq 0$) corresponds to the shell of $l = Q + n$ on a sphere.

Here, we do not use the particular form of the $|Q; n, m\rangle$ wavefunctions, but take advantage of the fact that the symmetry of angular momentum eigenstates $|l, m\rangle$ under 2D rotations mimics the symmetry of the planar eigenstates under 2D (magnetic) translations. Thus, the interaction matrix elements are guaranteed to obey general rules for a scalar operator in the basis of spherical harmonics, but the particular values are put into the model "by hand," so as to describe the actual interaction among the considered particles (on the plane). This is done by specifying Haldane pseudopotential,[28] defined as interaction energy V as a function of relative angular momentum $\mathcal{R}$. On a sphere relative and total pair angular momenta are related by $\mathcal{R} + L = 2l$, and the matrix elements $\langle l_1, m_1; l_2, m_2|V|l_3, m_3; l_4, m_4\rangle$ are connected with $V(L)$ through the Clebsch-Gordan coefficients.

The many-body interaction Hamiltonian is diagonalized numerically in the configuration-interaction basis, using a Lanczos algorithm. The energy levels E are determined separately for each subspace of the total spin S and angular momentum L.

III. SINGLE-QUASIELECTRON ENERGIES

In the mean-field CF transformation, the liquid of Laughlin correlated electrons at filling factor $\nu_e = \frac{4}{11}$ is converted to the system of CFs with an effective filling factor $\nu_{\mathrm{CF}} = \frac{4}{3}$. Thus, the low-energy dynamics of N_e electrons with Coulomb interaction in the lowest LL can be mapped onto that of $\sim \frac{3}{4}N_e$ CFs completely and rigidly filling the lowest CF-LL ($0\uparrow$) and the excess of $N \approx \frac{1}{4}N_e$ CFs in the $\nu = \frac{1}{3}$ filled next lowest CF-LL (either $1\uparrow$ or $0\downarrow$, depending on the relative magnitude of electron Zeeman energy E_{Z} and the effective CF cyclotron gap $\propto e^2/\lambda$). Each CF in the partially filled $1\uparrow$ or $0\downarrow$ LL represents a "normal"[2] or "reversed-spin"[15] quasielectron (QE or $\mathrm{QE_R}$) of the underlying incompressible Laughlin liquid, respectively.

The Coulomb energies $\varepsilon_{\mathrm{QE}}$ and $\varepsilon_{\mathrm{QER}}$ of these two QPs can be extracted[25,27] from exact diagonalization of finite systems of N_e electrons in the lowest LL with the appropriate degeneracy g. The Laughlin ground state occurs at $g = 3N_e - 2 \equiv g_{\mathrm{L}}$; it is non-degenerate ($L = 0$) and spin-polarized ($S = \frac{1}{2}N_e$). A single QE or $\mathrm{QE_R}$ appears in the Laughlin liquid in the lowest states at $g = g_{\mathrm{L}} - 1$ and either $S = \frac{1}{2}N_e$ or $\frac{1}{2}N_e - 1$, respectively. The QE and $\mathrm{QE_R}$ energies ε (defined relative to the underlying Laughlin liquid) are obtained from the comparison of the (N_e-electron) energies at $g = g_{\mathrm{L}}$ and and $g_{\mathrm{L}} - 1$.

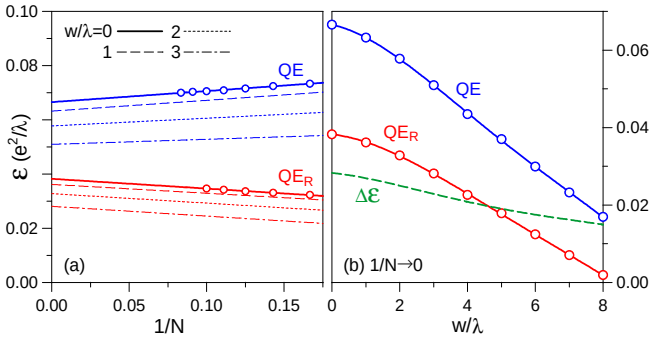

FIG. 1: (color online) Dependence of the quasielectron (QE) and reversed-spin quasielectron ($\mathrm{QE_R}$) energies ε on: (a) the inverse electron number N^{-1} in a finite-size calculation, and (b) the electron layer width w. λ is the magnetic length.

The numerical procedure and the result for an ideal 2D electron layer were presented earlier.[25,27] In Fig. 1 we compare the $\mathrm{QE/QE_R}$ energies calculated for quasi-2D layers of finite width w. Here, w is the effective width of the electron wavefunction in the normal (z) direction, approximated by $\chi(z) \propto \cos(z\pi/w)$. It is slightly larger than the quantum well width W; e.g., for symmetric $\mathrm{GaAs/Al_{0.35}Ga_{0.65}As}$ wells, $w \approx W + 3.3$ nm over a wide range of $W \geq 10$ nm. The regular dependence on system size in Fig. 1(a) allows reliable extrapolation of ε to $N^{-1} \to 0$ (planar geometry). From the comparison of $\varepsilon_{\mathrm{QE}}(w)$ and $\varepsilon_{\mathrm{QER}}(w)$ in Fig. 1(b) it is clear that their difference $\Delta\varepsilon$ is less sensitive to the width than any of the ε's. To put the shown width range in some perspective, let us note that a (fairly narrow) $W = 12$ nm well in a (fairly low) field $B = 10$ T corresponds to $w/\lambda = 1.9$ and $\Delta\varepsilon(w)/\Delta\varepsilon(0) = 0.9$, justifying the 2D approximation. On the other hand, a wide $W = 40$ nm well in a high field $B = 23$ T gives $w/\lambda = 8.1$ and $\Delta\varepsilon(w)/\Delta\varepsilon(0) = 0.5$, i.e., a significant width effect.

IV. QUASIELECTRON INTERACTIONS

The weak effective CF–CF interactions are known with some accuracy from earlier studies.[11,22,25,29,30] At least at sufficiently low CF fillings factors $\nu \leq \frac{1}{3}$, they can be well approximated by fixed Haldane pseudopotentials (independent of the CF-LL filling or spin polarization). The short-range QE–QE, $\mathrm{QE_R}$–$\mathrm{QE_R}$, and QE–$\mathrm{QE_R}$ pseudopotentials can be obtained from finite-size diagonalization for N_e electrons with up to two revesed spins ($S = \frac{1}{2}N_e - 2$) at $g = g_{\mathrm{L}} - 2$.

The result is a reliable account of the relative values $\Delta V_{\mathcal{R},\mathcal{R}'} = V(\mathcal{R}) - V(\mathcal{R}')$ at small neighboring $\mathcal{R}$ and $\mathcal{R}'$, but the absolute values are not estimated very accurately. Fortunately, since vertical correction of $V(\mathcal{R})$ by a constant does not affect the many-CF wavefunctions and only rigidly shifts the entire energy spectrum, a few leading values of ΔV completely determine the (short-range)

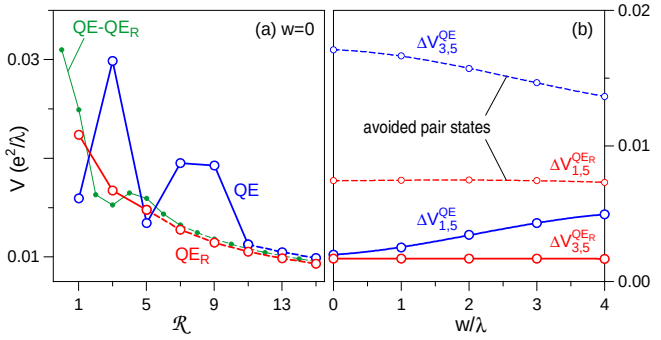

FIG. 2: (color online) (a) Haldane pseudopotentials (pair interaction energy V as a function of relative angular momentum $\mathcal{R}$) for quasielectrons (QE) and reversed-spin quasielectrons (QE$_R$) in an ideal 2D ($w = 0$) electron layer. (b) Dependence of pseudopotential increments $\Delta V_{\mathcal{R}\mathcal{R}'} = V(\mathcal{R}) - V(\mathcal{R}')$ on the electron layer width w. λ is the magnetic length.

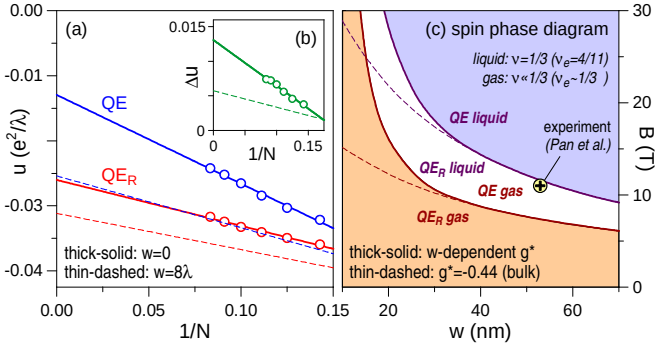

FIG. 3: (color online) (a) Correlation energy u in the $\nu = 1/3$ incompressible liquid of quasielectrons (QE) or reversed-spin quasielectrons (QE$_R$) as a function of their inverse number N^{-1}, for two different widths w of the quasi-2D electron layer (λ is the magnetic length). (b) Difference $\Delta u = u_{QE} - u_{QER}$ as a function of N^{-1}. (c) Phase diagram (critical layer width w vs magnetic field B) for the QE–QE$_R$ spin transition at $\nu = 1/3$ (i.e., $\nu_e = 4/11$), assuming effective electron g-factor for GaAs. Dashed line is for uncorrelated QEs or QE$_R$'s (e.g., at $\nu \ll 1/3$). The experimental point taken after Pan *et al.*[1]

CF correlations at a given ν. Therefore, the knowledge of those few approximate values of ΔV_{QER} and ΔV_{QE} was sufficient to establish that: (i) the QE$_R$'s form a Laughlin $\nu = \frac{1}{3}$ liquid[21,24,25] which in finite N-QE$_R$ systems on a sphere occurs at $g = 3N - 2$, and (ii) in contrast, the QEs form a different (probably paired) state[9,11] at the same $\nu = \frac{1}{3}$, which on a sphere occurs at $g = 3N - 6$.

However, the relative strength of QE–QE and QE$_R$–QE$_R$ pseudopotentials V_{QER} and V_{QE} must also be known (in addition to ΔV) to compare the energies of many-QE$_R$ and many-QE states (i.e., of the spin-polarized and unpolarized electron states at $\nu_e = \frac{4}{11}$). The absolute values of V_{QER} and V_{QE} can be obtained by matching[29] the short-range behavior from exact diagonalization of small systems with the long-range behavior predicted for a pair of charges $q = -\frac{1}{3}e$. Specifically, the short-range part of $V_{QER}(\mathcal{R})$, which describes a pair of CFs in the $1\downarrow$ CF-LL, is shifted to match $\eta V_0(\mathcal{R})$, the electron pseudopotential in the lowest LL rescaled by $\eta \equiv (q^2\lambda_q^{-1})/(e^2\lambda_e^{-1}) = (q/e)^{5/2}$. Similarly, the short-range part of $V_{QE}(\mathcal{R})$, related to the $1\uparrow$ CF-LL, is shifted to match $\eta V_1(\mathcal{R})$.

The result in Fig. 2(a) for an ideal 2D layer was reported earlier;[11] in Fig. 2(b) the width dependence of the leading parameters ΔV has been plotted. It is noteworthy that V_{QE} is much more sensitive to the electron layer width w than V_{QER}. This is explained by stronger oscillations in $V_{QE}(\mathcal{R})$ at $w = 0$, which tend to weaken in wider wells (when the characteristic in-plane distances decrease relative to w). The curves for $V_{QER}(1)$ and $V_{QE}(3)$ have been drawn with dashed lines, since the QE$_R$–QE$_R$ and QE–QE pair states associated with these dominant pseudopotential parameters will be avoided[9] in the unpolarized and polarized $\nu = \frac{1}{3}$ CF ground states, respectively.

V. CORRELATION ENERGIES OF QUASIELECTRON LIQUIDS

As mentioned above, due to the strong QE$_R$–QE$_R$ repulsion at short range ($\mathcal{R} = 1$), the QE$_R$'s form a Laughlin $\nu = \frac{1}{3}$ state similarly to the electrons at $\nu_e = \frac{1}{3}$. The corresponding series of non-degenerate N-QE$_R$ ground states on a sphere occurs at the Laughlin sequence of $g = 3N - 2$. In Fig. 3(a) we plot the size dependence of their correlation energy u (per particle), defined as

$$u = \frac{E + U_{bckg}}{N}\,\zeta. \qquad (1)$$

Here, E is the interaction energy of the ground state of N QE$_R$'s, $U_{bckg} = -(Nq)^2/2R$ is a correction due to interaction with the charge-compensating background (with the sphere radius $R = \lambda\sqrt{Q}$ taken for $2Q + 1 = g$, in analogy to the relation for electrons in the lowest LL). Factor $\zeta = \sqrt{Q(Q-1)^{-1}}$ is used to rescale the energy unit $e^2/\lambda = \sqrt{Q}\,e^2/R$ from that corresponding to $g_{QER} = 3N-2$ to that of an average $\bar{g} = \frac{1}{2}(g_{QER}+g_{QE}) = 3N-4$, to allow for a later comparison of u calculated for QE$_R$'s and QE's at different g's (and thus, at different magnetic lengths λ corresponding to the same area $4\pi R^2$).

The correlation energies u were calculated for $N \le 12$, and extrapolated to $N^{-1} \to 0$ to eliminate the finite-size effects. Neither the particular form of U_{bckg} (i.e., the assumption of $g = 2Q + 1$ for the relation between R and λ) nor the rescaling by ζ directly affect the extrapolated value (they only affect the size dependence, and thus the accuracy of extrapolation). For an ideal 2D system, the result of extrapolation is $u_{QER} = -0.026\,e^2/\lambda = -0.405\eta\,e^2/\lambda$. This value is very close to ηu_0, where $u_0 = -0.412\,e^2/\lambda$ describes the Laughlin state of electrons in LL$_0$. Good agreement confirms not only Laugh-

lin correlations among the QE_R's (which are guaranteed by the form of ΔV_{QER} and can also be verified directly by the analysis of pair amplitudes) but, more importantly, the accurate estimate of the absolute values of $V_{QER}(\mathcal{R})$ drawn in Fig. 2(a) and used in the N-QE_R calculation.

Let us turn to the QEs. The dominant QE–QE repulsion at $\mathcal{R} = 3$ causes the QEs to form pairs[11] rather than a Laughlin state at $\nu = \frac{1}{3}$ (although the exact wavefunction of this incompressible state is still unknown). The corresponding series of non-degenerate N-QE ground states on a sphere was identified[9] at $g = 3N - 6$, different from the Laughlin sequence. The QE correlation energy u was calculated from the same Eq. (1), but with a different $\zeta = \sqrt{Q(Q+1)^{-1}}$ (where also $g = 2Q + 1$). By using different ζ_{QER} and ζ_{QE} we removed discrepancy between λ/R of finite N-QE_R and N-QE systems, in order to improve size convergence of $\Delta u = u_{QE} - u_{QER}$.

In an ideal 2D system ($w = 0$), the extrapolated value at $N^{-1} = 0$ is $u_{QE} = -0.013\, e^2/\lambda$, twice smaller (in the absolute value) than u_{QER} of a Laughlin state. The difference $\Delta u = 0.013\, e^2/\lambda$ is the key numerical result of this paper. The accuracy of this estimate can be judged from the extrapolation plot in Fig. 3(b).

The fact that $u_{QER} < u_{QE}$ can be explained from the comparison[11] of QE_R and QE charge-density profiles $\rho(r)$. The roughly gaussian ρ_{QER} is (up to normalization) very similar to ρ_0 of an electron in the lowest LL, yielding similar QE_R and electron pseudopotentials $V(\mathcal{R})$ and correlation energies u (in the η-rescaled units). The ring-like ρ_{QE} is more complicated and has a bigger radius, causing stronger (on the average) QE–QE repulsion. The estimate of how much stronger – depends on the accurate matching of the short- and long-range QE–QE pseudopotentials in Fig. 2(a). Therefore, to gain more confidence, we compared u_{QER} with u_1 of the electrons filling $\nu = \frac{1}{3}$ of LL_1, whose ρ_1 falls between $\rho_{QER} \sim \rho_0$ and ρ_{QE} in terms of occupied area and the number of oscillations. For the known[31] $g = 3N - 6$ sequence of non-degenerate $\nu = \frac{1}{3}$ ground states in LL_1 we obtained $u_1 = -0.32\, e^2/\lambda$. Upon rescaling for the fractional QP charge, $\eta u_1 = -0.021\, e^2/\lambda$ falls between $\eta u_0 \approx u_{QER} = -0.026\, e^2/\lambda$ and $u_{QE} = -0.013$. This demonstrates that the difference between u_{QER} and u_{QE} is caused by the difference between ρ_{QER} and ρ_{QE}, and supports the obtained order of magnitude of Δu.

To demonstrate dependence of the correlation energies on layer width, in Figs. 3(a) and (b) we also showed data for $w = 8\lambda$. The extrapolated values for this very wide layer are $u_{QER} = -0.025\, e^2/\lambda$ and $u_{QE} = -0.031\, e^2/\lambda$. Significant decrease of both energies compared to $w = 0$ reflects an overall (averaged over in-plane distances, i.e., over $\mathcal{R}$) reduction of the QP repulsion in wider wells caused by the spread of electron (and thus also QE_R and QE) wavefunctions in the z-direction. Due to different in-plane dynamics, u_{QER} and u_{QE} depend differently on width, and their difference $\Delta u = 0.06\, e^2/\lambda$ at $w = 8\lambda$ is about twice smaller than at $w = 0$.

VI. SPIN PHASE DIAGRAM FOR $\nu_e = 4/11$

Whether QEs or QE_R's will form a $\nu = \frac{1}{3}$ state at $\nu_e = \frac{4}{11}$ depends on the competition of Coulomb and Zeeman energies. The condition for the QE↔QE_R transition is

$$\Delta\varepsilon + \Delta u = E_Z. \tag{2}$$

The competing phases differ in electron spin polarization ($P = 100\%$ vs 50%). They are both incompressible, but probably have different excitation gaps (and thus might not show equally strong FQH effect). In an ideal 2D electron layer, the excitation gap (for neutral excitations) of the polarized state can be expected[9] below $0.005\, e^2/\lambda$, and for the Laughlin state of QE_R's it is estimated at $\sim 0.06\eta\, e^2/\lambda = 0.004\, e^2/\lambda$ (note, however, that a much smaller value $\sim 0.001\, e^2/\lambda$ was predicted in Ref. 21). The nature of charged excitations, and the corresponding transport gaps (especially in more realistic conditions, i.e., for $w > 0$, including LL mixing and disorder, etc.) are not known, and their prediction should require a much more extensive calculation.

Let us concentrate on the question of stability of either QE_R's or QEs at $\nu_e = \frac{4}{11}$. In order to draw in Fig. 3(c) the phase diagram for GaAs heterostructures, we combined the estimated dependences of $\Delta\varepsilon/(e^2\lambda^{-1})$ and $\Delta u/(e^2\lambda^{-1})$ on w/λ (where $e^2\lambda^{-1}/\sqrt{B} = 4.49\,\mathrm{meV/T^{1/2}}$ and $\lambda\sqrt{B} = 25.6\,\mathrm{nm\,T^{1/2}}$) with published data[32] on width dependence of the effective Landé factor g^*, governing the Zeeman splitting $E_Z = g^*\mu_B B$ (for $W \geq 30$ nm, it is $g^* = -0.44$ and $E_Z/B = 0.03$ meV/T; in narrower wells, g^* increases, passing through zero at $W \approx 5.5$ nm; recall that $w \approx W + 3.3$ nm).

The most important phase boundary drawn in Fig. 3(c) divides the polarized and unpolarized $\nu_e = \frac{4}{11}$ states, i.e., the correlated QE and QE_R liquids at a finite $\nu = \frac{1}{3}$. In experiment of Pan *et al.*[1] the polarized $\nu_e = \frac{4}{11}$ state was observed in a symmetric $W = 50$ nm GaAs quantum well at $B = 11$ T. The corresponding point (w, B) lies very close to predicted phase boundary, suggesting that the experimentally detected polarization depended critically on the choice of a very wide well. It is clear from Fig. 3(c) that the spin transition in narrower wells shifts quickly to higher magnetic fields (i.e., to higher electron concentrations $\varrho_e = \nu_e(2\pi\lambda^2)^{-1}$), especially when the width dependence of g^* is taken into account. This suggests that the spin transition at $\nu_e = \frac{4}{11}$ might be confirmed in a similar experiment, carried out in a sample with the same W and ϱ_e, but with the layer width w tuned by the electric gates (inducing a controlled well asymmetry).

The role of QP interaction in stabilizing the QE_R phase is clear from the comparison of boundaries dividing correlated QE/QE_R liquids and non-interacting QE/QE_R gases (the gas occurs at $\nu \ll \frac{1}{3}$, with the critical equation $\Delta\varepsilon = E_Z$; the CF gas↔liquid transition was recently demonstrated by inelastic light scattering[33]). Additional boundaries (not shown here, but cf. Fig. 13(b) in Ref. 20) appear at even smaller B, defining the areas of stabil-

ity for a gas of CF skyrmions of different sizes.[18,19,20,34] Note also that $\Delta\varepsilon$ is determined more accurately than Δu, possibly explaining the incorrect position of the experimental point inside the predicted QE-gas/QE_R-liquid area.

VII. CONCLUSION

Combining composite fermion theory with exact numerical diagonalization we studied two spin states of the "second-generation" incompressible quantum liquid at $\nu_e = \frac{4}{11}$. Our main result is prediction of a transition between these competing states, different not only by the spin polarization, but also by the microscopic mechanism of incompressibility (the nature of CF–CF correlation). Starting with effective interaction pseudopotentials of polarized and reversed-spin Laughlin quasielectrons (QE and QE_R), we determined their correlation energies u in conditions adequate for realistic 2D electron layers of different widths w and in different magnetic fields B. This allowed us to draw a spin phase diagram of the $\nu_e = \frac{4}{11}$ state in the (w, B) coordinates. Comparison of our numerics with the experiment of Pan *et al.* is not conclusive. However, our prediction of the spin transition induced in the same quantum well by external electric gates offers a possibility of more accurate testing of the theory. Finally, we have only considered pure QE or QE_R states (i.e., confined ourselves to the extreme polarizations of $P = 100\%$ and 50% in constructing the $\nu_e = \frac{4}{11}$ phase diagram), leaving out the possibility of mixed QE/QE_R states with intermediate P near the predicted phase boundary.

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